Volume Refinement Fairing Isosurfaces

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Overview

• Motivation
• Continuous Approach
• Discretization
• Results

Motivation

• low-resolution data
• interpolation artifacts

Refinement + Fairing Isolines

bilinear / bicubic
our method
Idea

• fairing scalar fields by fairing contours
• interpolation at grid points
• refinement
  – dof’s for optimization
  – enhance features
  – reduce artifacts
  – unknown contour topology

Related Work

• isosurfaces
  – extraction with trilinear topology
    [Lopes, Brodlie 93], [Nielson 93]
  – simplification
• fairing
  – fitting with splines / wavelets
  – variational modeling
  – anisotropic diffusion

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Optimization

• interpolation constraints $f(s_i, t_i) = p_i$
• residual $r := < f, f > \rightarrow \min$
  $< f, g >_{TP} := \iint f_{ss} g_{ss} + 2f_{st} g_{st} + f_{tt} g_{tt} dsdt$
• system of equations for dof’s $\frac{\partial r}{\partial c_j} = 0$
(1) Approximation

\[(1 - w) \sum_i |f(s_i, t_i)|^2 + w <f, f> \to \min\]

least squares "energy"

system

\[\left((1 - w)2A^T A + wE\right) c = (1 - w)2A^T p\]

\[a_{ij} = \psi_j(s_i, t_i), \quad e_{ij} = <\psi_i, \psi_j>\]

(2) Interpolation

- 2 sets of basis functions
  - interpolation \(\phi_i(s_j, t_j) = \delta_{ij}\)
  - optimization \(\psi_k(s_j, t_j) = 0\)

\[Ec = Gp, \quad e_{ij} = <\psi_i, \psi_j>, \quad g_{ij} = <\psi_i, \varphi_j>\]

Fairing Isolines

- parametric isoline \(g(s)\) with normal \(n(s)\)
- minimize variation of \(n\)

\[\int \|n \times \text{grad} \ n\|^2 \ ds \to \min\]

\[\int \|n \times \text{grad} \ n\|^2 \ ds \to \min\]

(for all isovalues \(\alpha\))

\[\int \int \|n \times \text{grad} \ n\|^2 \ dsd\alpha \to \min\]

\[\int \int \|n \times \text{grad} \ n\|^2 \ dx dy \to \min\]

(non-linear optimization problem)

\[<f, f>_n \to \min\]
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Discrete Residual

- piece-wise linear representation

\[ r := \sum_{a,b \text{adj}} w_{ab} (n_a - n_b)^2 \rightarrow \min \]

\[ w_{ab} = (e_{ab} \cdot (n_a + n_b)/2)^2 \]

Discrete Residual

- local residual \( r = r(f_{ij}) \) is a quartic polynomial
- iteration:
  for all \( i,j \) find
  \( f_{ij}: r(f_{ij}) \rightarrow \min \n
Triangulation

triangulation artifacts

optimize triangles
Algorithm

- refinement (trilinear)
- iteration on 3 sets of slices
- for every vertex
  minimize local residual

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Results

bilinear 1st iteration 10 iterations refinement

bilinear 5 iterations 2nd refinement
Computation Times

<table>
<thead>
<tr>
<th>data set</th>
<th>res.</th>
<th># dofs</th>
<th>t / iter. [sec]</th>
</tr>
</thead>
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<tr>
<td>m.lobb</td>
<td>$81^3$</td>
<td>0.46M</td>
<td>22.9</td>
</tr>
<tr>
<td>skull</td>
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<td>1.79M</td>
<td>85.3</td>
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<td>m.lobb</td>
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<tr>
<td>skull</td>
<td>$253^3$</td>
<td>14.15M</td>
<td>676.0</td>
</tr>
</tbody>
</table>

Conclusions

- non-linear subdivision
- variational principles
- fairing all contours in one pass
- $O(n)$ (but still slow)
- may be used locally
Future Work

- optimal basis functions
- efficiency
- 3D algorithm
- non-uniform grids